if $r''(n)$ denotes the number of primitive representations of n by the form $x^2 + 3y^2 + 3z^2$, then $r'(n) = k \cdot r''(n)$, where $k = 1$ if $n \equiv 19 \pmod{24}$, $k = 3$ if $n \equiv 1$ or $10 \pmod{12}$; and if $9 \nmid n$, $k = 2$ if $n \equiv 3 \pmod{24}$, $k = 6$ if $n \equiv 6$ or $9 \pmod{12}$; for other linear forms of n, either $r'(n)$ or $r''(n)$ is 0.

The Equivalence of Bilinear Forms

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The equivalence problem of nondegenerate bilinear forms (no "symmetry" assumption whatsoever) was largely solved by J. Williamson over 40 years ago; more recently, G. E. Wall extended these results. I have recast this theory in what seems to me to be a clearer and more useful form, along lines used by J. Milnor and others in the closely related theory of conjugacy in the classical groups. Furthermore this method leads to a solution of the only case no handled previously, the "unipotent" case in characteristic 2. This is very much an exceptional case, the additional difficulties being similar to those in O. T. O'Meara's theory of integral quadratic forms over dyadic local fields.

The Factorization of an Integral Matrix into a Product of Two Integral Symmetric Matrices. I

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It is known that every n by n matrix A can be factorized into a product of two symmetric matrices $A = S_1S_2$ with elements in the field of the elements of A.

It seems natural to ask under what conditions the elements of S_1 and S_2 can be chosen in a ring R if A has its elements in R. Here the case where R is the ring of rational integers is studied. It will be assumed that A has an irreducible characteristic polynomial $f(x)$ over Q. It is known that A is then associated with an ideal class C in $Z[\alpha]$ where α is a zero of $f(x)$. If $Z[\alpha]$ is the maximal order in $Q(\alpha)$ then S_1 or S_2 can even be chosen unimodular provided C is of order 1 or 2 in the ideal class group. This follows from results of Fadeev and Taussky.

The case $n = 2$ is studied here[1] for $f(x) = x^2 - m$, m squarefree, $m \equiv 2$ or $3(4)$. A necessary and sufficient condition for the integral factorization of A is then that C has order 1 or 2 or 4, with some exceptional cases for 4 which can be characterized easily. The problem leads to associating an integral binary quadratic form $\mathfrak{a}(\lambda, \mu)$ of discriminant $4m$ with A, and the factorization depends on whether a factor of m is represented by this form. This forces the form to be in a class of order 1 or 2.

It is further shown that the ideal class to which $\mathfrak{a}(\lambda, \mu)$ corresponds is the square of C^{-1}. Using previous results by the author it follows that it is connected with the class of the ideal $(\mathfrak{A}' : \mathfrak{A})$ in the general case when $\mathfrak{A} \in C$ and $\mathfrak{A}'$ is the complementary ideal of $\mathfrak{A}$. Use is made of the fact that for $n = 2$ every rational matrix S for which $S^{-1}AS$ is the transpose of A has as determinant a negative norm in $Q(\alpha)$.

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[1] The general 2 by 2 case has been settled in the meantime, leading to the result that every integral 2 by 2 matrix can be "factorized" when a suitable integral scalar matrix is added to it.